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Engwerda, J.C.

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BRABANT

POSTBOX 90153
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LQ-PROBLEM: THE DISCRETE-TIME
TIME-VARYING CASE

J.C. Engwerda

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LQ-PROBLEM: THE DISCRETE-TIME TIME-VARYING CASE

Abstract

In this paper we solve the Linear Quadratic (LQ) regulator problem for discrete-time time-varying systems. By making an appropriate state-space decomposition of the system, sufficient conditions are derived under which this LQ-problem is solvable and, moreover, the closed-loop system becomes exponentially stable.

These conditions are extensions of the time-invariant notions of stabilizability and detectability. Unfortunately, in general these conditions are not necessary. The approach we take provides, however, also a good insight into the difficulties that occur if one looks for both necessary and sufficient conditions solving the problem.

Keywords

Linear discrete-time time-varying systems, stabilizability, detectability, state-space decomposition.

I. INTRODUCTION

In the past much research has been done on the subject under which conditions the linear quadratic regulator problem has a solution if the considered system is time-varying, see e.g. Kwakernaak et al (1971), Hager et al (1976) and Anderson et al (1980, 1981).

In Anderson et al (1981) it was claimed that under a uniform stabilizability and uniform detectability condition the Kalman filter, the dual of the LQ problem, is exponentially stable (under the usual system noise assumptions). Engwerda showed by means of a counterexample in (1988-a), however, that this claim is not correct. In fact he showed that the definitions given by Anderson et al of uniform stabilizability and uniform detectability do not imply that the system is stabilizable and detectable, respectively.

For that reason Engwerda formulated in the same paper new conditions which imply (exponential) stabilizability and detectability of the system. These new conditions are formulated in terms of a transformed system that is obtained by applying an appropriate state-space decomposition. In this paper we use and extend this analysis in order to obtain sufficient conditions under which the LQ-problem has a solution with the property that it makes the closed-loop system exponentially stable. The paper is organized as follows.

First, we introduce in section 2 the notions of uniform periodic smooth exponential stabilizability and detectability respectively. Then, we show in section 3 that under these conditions the LQ problem has a solution. Consecutively, we show that when the resulting optimal state-feedback control is applied the system becomes exponentially stable. The paper ends with some concluding remarks.

II. PRELIMINARIES

As announced in the previous section we introduce here the notions of uniform periodic smooth exponential stabilizability and detectability. To that extent we first state the considered system and some definitions. We will be dealing with the following linear time-varying discrete-time system:

$$\Sigma_y: \begin{aligned} x(k+1) &= A(k)x(k) + B(k)u(k); \quad x(k_0) = \bar{x} \\ y(k) &= C(k)x(k), \end{aligned}$$

where $x(k) \in \mathbb{R}^n$ is the state of the system, $u(k) \in \mathbb{R}^m$ the applied control and $y(k) \in \mathbb{R}^r$ the output at time k . Moreover, we assume that all matrices $A(\cdot)$, $B(\cdot)$ and $C(\cdot)$ are bounded.

Since the system is time-varying it is convenient to have the notation:

Let N be any positive number, then

$$\begin{aligned} A(k+N, k) &:= A(k+N-1) \dots A(k) \quad \text{if } N \geq 1 \\ &:= I \quad \text{if } N=1 \end{aligned}$$

$$S[k, k-N] := [B(k) | A(k+1, k)B(k-1) | \dots | A(k+1, k-N+1)B(k-N)]$$

$$W[k, k-N] := [C(k)^T | \dots | \{C(k+N)A(k+N, k)\}^T]^T$$

$$v[k, \ell] := (v^T(k), \dots, v^T(\ell))^T$$

$$v[k, \cdot] := (v^T(k), v^T(k+1), \dots)^T$$

$x(k, k_0, \bar{x}, u)$ is the state of the system at time k resulting from the

initial state \bar{x} at time k_0 if the input $u[k_0, k-1]$ is applied

$$y(k, k_0, \bar{x}, u) := C(k)x(k, k_0, \bar{x}, u). \quad \square$$

Using this notation we can give now easily formal definitions of several notions that are used later on in this paper.

Definition 1

The initial state \bar{x} of the system Σ_y is said to be

- * exponentially stable at k_0 if there exist positive constants α and M such that $\|x(k, k_0, \bar{x}, 0)\| \leq M e^{-\alpha(k-k_0)} \|\bar{x}\|$ for any $k > k_0$
- * exponentially stabilizable at k_0 if there exists a control sequence $u[k_0, \cdot]$, with the property that $u(\cdot)$ converges exponentially fast to

zero, and positive constants α and M such that

$$\|x(k, k_0, \bar{x}, u)\| \leq M e^{-\alpha(k-k_0)} \|\bar{x}\| \text{ for any } k \geq k_0.$$

- * unobservable at k_0 if $y(k, k_0, \bar{x}, 0) = 0$ for any $k \geq k_0$.
- * exponentially detectable at k_0 if there exists a finite integer $N > 0$ such that \bar{x} modulo $X_e^-(A(\cdot, k_0))$ is determined from any $y[k_0, k_0 + N - 1]$ and $u[k_0, k_0 + N - 2]$. Here $X_e^-(A(\cdot, k_0))$ is the linear subspace consisting of all exponentially stable states at time k_0 .

Like all exponentially stable states, the set of all unobservable states at k_0 constitute a linear subspace. We denote it by U_{k_0} . Now, Σ_y is called observable at k_0 if $x=0$ is the only unobservable state at k_0 . Moreover, we say that Σ_y is exponentially stable (respectively stabilizable, exponentially detectable) at k_0 if any initial state of Σ_y possesses the corresponding property at k_0 .

Using these concepts the notion of uniform exponential stabilizability and detectability are defined as follows.

Definition 1 (continued):

Σ_y is called uniformly exponentially stabilizable (respectively detectable) if Σ_y has the corresponding property at any time $k \geq k_0$ and, moreover, the constants $\alpha(k)$ and $M(k)$ appearing in the definitions satisfy the inequalities $\alpha(k) > \bar{\alpha} > 0$ and $M(k) < \bar{M} < \infty$ for some $\bar{\alpha}$ and \bar{M} . □

Another notion that plays an important role in our analysis is the concept of reachability. Formally, we call a state \bar{x} reachable (from zero) if there exists a control sequence $u[N, k-1]$ with $-\infty < N < k$ such that $x(k, N, 0, u) = \bar{x}$. The linear subspace of all reachable states at time k is denoted by R_k .

Now, Engwerda showed in (1987) that R_k is $A(k)$ -invariant, that is, $A(k)R_k \subset R_{k+1}$. Moreover, he showed in (1988-b) that the unobservability subspace has this property too. These properties are used in lemma 2, where we give an equivalent system representation of Σ_y . To that extent we first introduce the state-space decomposition

$$X_1(k) = R_k \cap U_k;$$

$$\begin{aligned} X_1(k) \otimes X_2(k) &= R_k; \\ X_1(k) \otimes X_2(k) \otimes X_3(k) &= R^n, \end{aligned}$$

where X_1 , X_2 and X_3 are chosen orthogonal. According to Engwerda (1988-a) the next result holds

Lemma 2

There exists an orthogonal state-space transformation $x(.) = T'(.)x'(.)$, which does not effect the boundedness property of the system parameters, such that Σ_y is described by the recurrence equation.

$$\Sigma_y': \begin{array}{l} x_1'(k+1) \\ x_2'(k+1) \\ x_3'(k+1) \end{array} = \begin{array}{ccc} A_{11}'(k) & A_{12}'(k) & A_{13}'(k) \\ 0 & A_{22}'(k) & A_{23}'(k) \\ 0 & 0 & A_{33}'(k) \end{array} \begin{array}{l} x_1'(k) \\ x_2'(k) \\ x_3'(k) \end{array} + \begin{array}{l} B_1'(k) \\ B_2'(k) \\ B_3'(k) \end{array} u'(k)$$

$$y(k) = (0 \quad C_1'(k) \quad C_2'(k))x'(k),$$

where

$$\Sigma_1': x_1'(k+1) = A_{11}'(k)x_1'(k) + B_1'(k)u'(k) \text{ is reachable at any time } k \geq k_0.$$

$$\Sigma_2': x_2'(k+1) = A_{22}'(k)x_2'(k) + B_2'(k)u'(k) + A_{23}'(k)A_{33}'(k+1, k_0)x_3'(k_0)$$

$$y(k) = C_2'(k)x_2'(k) \text{ is both reachable and observable at any time } k \geq k_0;$$

$$\Sigma_3': x_3'(k+1) = A_{33}'(k)x_3'(k).$$

□

In order to obtain sufficient conditions for exponential stabilizability and detectability of Σ_y at k_0 we introduce the notions of periodic smooth controllability and observability. Roughly spoken, we say that a system is periodically smoothly controllable if there exists a finite time period such that whenever such a time period has passed, the system has been at least once controllable during that period.

Definition 3.

Σ_y is called periodically smoothly controllable at k_0 if there exist positive constants ϵ and k_1 such that for all $k > 0$ there exists an integer $k_2(k)$ in the interval $[k_0 + (k-1)k_1, k_0 + k k_1]$ for which $S[k_2-2 k_1, k_2]S^T[k_2-2 k_1, k_2] \geq \epsilon I$.

Similarly we say that Σ_y is periodically smoothly observable at k_0 if there exist positive constants b and k_1 such that for all $k > 0$ there exists an integer $k_2(k)$ in the interval $[k_0 + (k-1)k_0, k_0 + k k_0]$ for which $W[k_2, k_2 + 2k_1] W^T[k_2, k_2 + 2k_1] \geq b I$.

Instead of periodic smooth controllability (respectively observability) of Σ_y we often use the phraseology periodic smooth controllability of the pair $(A(\cdot), B(\cdot))$ and observability of the pair $(C(\cdot), A(\cdot))$, respectively. □

With the notation of lemma 2, we then have as a special case from theorem 20 of Engwerda (1988-a):

Theorem 4:

Σ_y is both exponentially stabilizable and exponentially detectable at k_0 if the following three conditions are satisfied:

- i) Σ'_1 is uniformly exponentially stable;
- ii) Σ'_2 is both periodically smoothly controllable and observable at k_0 ;
- iii) Σ'_3 is exponentially stable at k_0 .

In the next section we will need that Σ_y is both uniformly exponentially stabilizable and uniformly exponentially detectable. From theorem 4 it is clear that this property holds if additional to the conditions i) and ii), Σ'_3 is uniformly exponentially stable. We state this result in a corollary.

Corollary 5:

Σ_y is both uniformly exponentially stabilizable and uniformly exponentially detectable if

- i) Σ'_1 is uniformly exponentially stable
- ii) Σ'_2 is both periodically smoothly controllable and observable at k_0
- iii) Σ'_3 is uniformly exponentially stable.

In the sequel we call conditions i) upto iii) in corollary 5 the exponential stabilizability and detectability (E.S.D.) conditions. Note that for time-invariant systems these three conditions are necessary too.

III The solution of the LQ control problem.

In this section we consider the LQ optimal control problem:

$$(1) \quad \min_{u[k_0, \dots]} \lim_{N \rightarrow \infty} J_N, \text{ subject to } \sum y$$

where

$$J_N = \sum_{k=k_0}^{k_0+N-1} \{ \|y(k)\|^2 + \|u(k)\|_{R(k)}^2 \} + \|y(k_0+N)\|^2.$$

In the sequel we take without loss of generality $k_0 = 0$. Moreover shall $C^T C$ be denoted by Q .

Furthermore, we assume that the following, the so-called Sufficient Control Existence (S.C.E.), conditions are satisfied.

i) The E.S.D. conditions of corollary 5
(S.C.E.)

ii) a) $R(k) \geq \beta I$ for some $\beta > 0$, for all $k \geq 0$

or b) $B^T(k)Q(k+1)B(k) \geq \beta_1 I$, $Q(k) \geq \beta_2 I$ and $R(k) \geq 0$ for some $\beta_i > 0$, $i = 1, 2$ for all k .

We will show that under these conditions an optimal control for the LQ problem exists and is given by:

$$(2) \quad u(k) = -F(k) x(k)$$

where $F(k) = (R(k) + B^T(k)K(k+1)B(k))^{-1} B^T(k)K(k+1)A(k)$,
and $K(k)$ is given by

$$K(k) = \lim_{N \rightarrow \infty} K_N(k), \text{ where } K_N(k) \text{ is obtained from the}$$

recursive equation:

$$(RRE): K_N(k) = A^T(k) \{ K_N(k+1) - K_N(k+1)B(k)(R(k) + B^T(k)K_N(k+1)B(k))^{-1} \cdot$$

$$B^T(k)K_N(k+1)\}A(k) + Q(k), K_N(N) = Q(N).$$

Moreover, we show that if this optimal feedback controller (2) is used to regulate the system, the closed-loop system becomes exponentially stable.

Theorem 6:

Let Σ_y satisfy the S.C.E. conditions.

Then, controller (2) minimizes $\lim_{N \rightarrow \infty} J_N$.

Proof:

First, consider the optimal control problem $\min_{u[0, N-1]} J_N$ subject to Σ_y .

The optimal control for this problem is:

$$(3) \quad u_N(k) = -F_N(k) x(k)$$

where $F_N(k) = (R(k) + B^T(k)K_N(k+1)B(k))^{-1} B^T(k)K_N(k+1)A(k)$, and $K_N(k)$ is given by the resursive equation (RRE).

Moreover, we have that the corresponding minimal control cost equals $\bar{J}_N := x^T(0)K_N(0)x(0)$ (see e.g. Bertsekas (1976)).

Since, due to our assumptions, Σ_y is exponentially stabilizable we have that there exists a control sequence such that $\lim_{N \rightarrow \infty} J_N$ remains finite.

Now, \bar{J}_N is a monotonically increasing sequence.

Consequently, $\lim_{N \rightarrow \infty} K_N(0)$ exists. Moreover, since Σ_y is uniformly exponentially stabilizable, a similar reasoning shows that $\lim_{N \rightarrow \infty} K_N(k)$ exists for any k .

So, we have shown now that $\lim_{N \rightarrow \infty} u_N(k)$ exists. Denote this limit by $\bar{u}(k)$.

Due to the monotonicity property of \bar{J}_N we can apply Bellman's principle to conclude that

$$\lim_{N \rightarrow \infty} \bar{J}_N \leq \min_{u[0, .]} \lim_{N \rightarrow \infty} J_N.$$

So, the only thing left to be proved is that

$$\lim_{N \rightarrow \infty} \bar{J}_N \geq \min_{u[0, .]} \lim_{N \rightarrow \infty} J_N.$$

This can be done by using some elementary analysis. Since J_N consists of the sum of positive functions, Fatou's lemma (see Rudin (1976), Theorem 11.31) can namely be applied to conclude that the order of taking limits and summations can be interchanged (for a more detailed proof see Engwerda (1988-b)). This completes the proof. \square

To prove exponential stability of $x(k+1) = (A-BF)(k)x(k)$ we need an extended result of Lyapunov's lemma. This result can be proved along the lines the proof of the corresponding property for uniformly stabilizable and detectable systems in Anderson et al (1981) (see Engwerda (1988-b) for a detailed proof).

Lemma 7: (Extended lemma of Lyapunov).

Let $A(\cdot)$ and $H(\cdot)$ be bounded.

Suppose that $(A(\cdot), H(\cdot))$ is periodically smoothly observable and that there is a bounded positive semi-definite symmetric matrix sequence $P(k)$ satisfying $A^T(k)P(k+1)A(k) - P(k) = -H^T(k)H(k)$ on $[0, \infty)$.

Then $x(k+1) = A(k)x(k)$ is exponentially stable. \square

We are now able to prove the main result of this paper.

Theorem 8:

Let the S.C.E. conditions be satisfied.

Then there exist constants $M < \infty$ and $\alpha > 0$ such that $\|(A-BF)(k,0)\| \leq Me^{-\alpha k}$.

Proof:

We know from theorem 6 that we can associate the following control problem with $(A-BF_N)(\cdot)$:

$$\min_{u[0, N-1]} J_N, \text{ subject to } \Sigma_y.$$

We reconsider this minimization problem.

From lemma 2 we have that this problem can be rewritten as:

$$\min_{u'[0, N-1]} \sum_{k=0}^{N-1} \{ \|x'_2(k)\|_{C_2^T(k)C_2'(k)}^2 + \|x'_3(k)\|_{C_3^T(k)C_3'(k)}^2 +$$

$$\|u'(k)\|_{R(k)}^2 + \|x'_2(N)\|_{C_2'^T(N)C_2'(N)} + \|x'_3(N)\|_{C_3'^T(N)C_3'(N)}^2,$$

subject to Σ_y .

According to the proof of theorem 6 the optimal control is given by (3). Substitution of the system parameters yields (by induction on k) that $K'_N(k)$ has the following structure:

$$K'_N(k) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & K_{22} & K_{23} \\ 0 & K_{23} & K_{33} \end{bmatrix}_N'(k) \quad (i)$$

and consequently

$$F'_N(k) = (0 \mid F_2 \mid F_3)_N'(k) \quad (ii)$$

Since $K'_N(\cdot)$ converges to $K'(\cdot)$ and $F'_N(\cdot)$ to $F'(\cdot)$ it is clear that $K'(\cdot)$ and $F'(\cdot)$ have the structure of (i) and (ii), respectively. Since $K'(\cdot)$ converges for any k we have from (RRE), moreover, that $K'(\cdot)$ satisfies the recurrence equation

$$K'(k) = A'^T(k) \{ K'(k+1) - K'(k+1)B'(k)(R(k) + B'^T(k)K'(k+1)B'(k))^{-1} B'^T(k)K(k+1) \} A'(k) + Q'(k),$$

which can be rewritten as:

$$K'(k) = (A-BF)^T(k) K'(k+1) (A-BF)'(k) + (Q+F^TRF)'(k).$$

In particular it follows now, by substitution of all the system parameters, that

$$K'_{22}(k) = (A_{22}-B_2F_2)^T(k) K'_{22}(k+1) (A_{22}-B_2F_2)'(k) + (C_2^TC_2 + F_2^TRF_2)'(k)$$

with

$$F_2'(k) = (R(k) + B_2'^T(k)K'_{22}(k+1)B_2'(k))^{-1} B_2'^T(k)K'_{22}(k+1)A'_{22}(k).$$

From the S.C.E. conditions it follows that $K'_{22}(\cdot)$ and $F'_2(\cdot)$ are bounded. Now, let

$$D' := (C_2'^T \mid F_2'^T R^{\frac{1}{2}})^T$$

then,

$$A'_{22} - B_2' F_2' = A'_{22} - [0 \mid B_2'] R^{-\frac{1}{2}} D' \text{ and } C_2'^T C_2' + F_2'^T R F_2' = D'^T D'.$$

Since the observability property of (A'_{22}, C_2') implies that (A'_{22}, D') has the same property, it is easily shown that $(A'_{22} - B_2' F_2', D')$ is periodically smoothly observable too (see e.g. Anderson (1981) or Engwerda (1988-b)). Application of lemma 7 yields now that $(A'_{22} - B_2' F_2')'(\cdot)$ is exponentially stable.

Since the feedback gain F' does not influence the exponential stability of Σ'_1 and Σ'_3 , we conclude that the overall CL-system, $(A - BF)'(\cdot)$, is exponentially stable.

Finally, we note that since the transformation matrix $T'(k)$ is bounded, exponential convergence of $x'(k)$ implies that the same property holds for the original state of the system $x(k)$. Which completes the proof. \square

Concluding Remarks.

In this paper we solved the discrete-time time-varying LQ optimal control problem under some weak conditions on the system. These conditions were formulated in terms of a transformed system that was obtained by making use of several invariance properties of the system.

A major problem occurring was to find a suitable state-space representation. This, since the prerequisite that the convergence properties of the transformed and original system must coincide, reduces the class of admissible transformations.

Fortunately, we succeeded in finding such a transformation which, moreover, was useful when we had to prove that the closed-loop system is exponentially stable if the optimal LQ control is applied.

An interesting question which remains to be solved is whether the LQ-problem has also an exponentially stabilizing solution when the system is both uniformly exponentially stabilizable and detectable (in the sense defined here). A subsequent question, which immediately arises if the answer to the previous one is affirmative, is then to give both necessary and sufficient conditions (that can a priori be verified) that guarantee this uniform exponential stabilizability and detectability.

Last, but not least, we note that the obtained results can be used in a straightforward manner to solve e.g. the LQG-problem, the EQL-problem (see Engwerda (1988-b)) and the Kalman filter problem for discrete-time time-varying systems.

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IN 1987 REEDS VERSCHENEN

- 242 Gerard van den Berg
Nonstationarity in job search theory
- 243 Annie Cuyt, Brigitte Verdonk
Block-tridiagonal linear systems and branched continued fractions
- 244 J.C. de Vos, W. Vervaat
Local Times of Bernoulli Walk
- 245 Arie Kapteyn, Peter Kooreman, Rob Willemse
Some methodological issues in the implementation
of subjective poverty definitions
- 246 J.P.C. Kleijnen, J. Kriens, M.C.H.M. Lafleur, J.H.F. Pardoel
Sampling for Quality Inspection and Correction: AOQL Performance
Criteria
- 247 D.B.J. Schouten
Algemene theorie van de internationale conjuncturele en structurele
afhankelijkheden
- 248 F.C. Bussemaker, W.H. Haemers, J.J. Seidel, E. Spence
On (v,k,λ) graphs and designs with trivial automorphism group
- 249 Peter M. Kort
The Influence of a Stochastic Environment on the Firm's Optimal Dynamic
Investment Policy
- 250 R.H.J.M. Gradus
Preliminary version
The reaction of the firm on governmental policy: a game-theoretical
approach
- 251 J.G. de Gooijer, R.M.J. Heuts
Higher order moments of bilinear time series processes with symmetrically
distributed errors
- 252 P.H. Stevers, P.A.M. Versteijne
Evaluatie van marketing-activiteiten
- 253 H.P.A. Mulders, A.J. van Reeken
DATAAL - een hulpmiddel voor onderhoud van gegevensverzamelingen
- 254 P. Kooreman, A. Kapteyn
On the identifiability of household production functions with joint
products: A comment
- 255 B. van Riel
Was er een profit-squeeze in de Nederlandse industrie?
- 256 R.P. Gilles
Economies with coalitional structures and core-like equilibrium concepts

- 257 P.H.M. Ruys, G. van der Laan
Computation of an industrial equilibrium
- 258 W.H. Haemers, A.E. Brouwer
Association schemes
- 259 G.J.M. van den Boom
Some modifications and applications of Rubinstein's perfect equilibrium model of bargaining
- 260 A.W.A. Boot, A.V. Thakor, G.F. Udell
Competition, Risk Neutrality and Loan Commitments
- 261 A.W.A. Boot, A.V. Thakor, G.F. Udell
Collateral and Borrower Risk
- 262 A. Kapteyn, I. Woittiez
Preference Interdependence and Habit Formation in Family Labor Supply
- 263 B. Bettonvil
A formal description of discrete event dynamic systems including perturbation analysis
- 264 Sylvester C.W. Eijffinger
A monthly model for the monetary policy in the Netherlands
- 265 F. van der Ploeg, A.J. de Zeeuw
Conflict over arms accumulation in market and command economies
- 266 F. van der Ploeg, A.J. de Zeeuw
Perfect equilibrium in a model of competitive arms accumulation
- 267 Aart de Zeeuw
Inflation and reputation: comment
- 268 A.J. de Zeeuw, F. van der Ploeg
Difference games and policy evaluation: a conceptual framework
- 269 Frederick van der Ploeg
Rationing in open economy and dynamic macroeconomics: a survey
- 270 G. van der Laan and A.J.J. Talman
Computing economic equilibria by variable dimension algorithms: state of the art
- 271 C.A.J.M. Dirven and A.J.J. Talman
A simplicial algorithm for finding equilibria in economies with linear production technologies
- 272 Th.E. Nijman and F.C. Palm
Consistent estimation of regression models with incompletely observed exogenous variables
- 273 Th.E. Nijman and F.C. Palm
Predictive accuracy gain from disaggregate sampling in arima - models

- 274 Raymond H.J.M. Gradus
The net present value of governmental policy: a possible way to find the Stackelberg solutions
- 275 Jack P.C. Kleijnen
A DSS for production planning: a case study including simulation and optimization
- 276 A.M.H. Gerards
A short proof of Tutte's characterization of totally unimodular matrices
- 277 Th. van de Klundert and F. van der Ploeg
Wage rigidity and capital mobility in an optimizing model of a small open economy
- 278 Peter M. Kort
The net present value in dynamic models of the firm
- 279 Th. van de Klundert
A Macroeconomic Two-Country Model with Price-Discriminating Monopolists
- 280 Arnoud Boot and Anjan V. Thakor
Dynamic equilibrium in a competitive credit market: intertemporal contracting as insurance against rationing
- 281 Arnoud Boot and Anjan V. Thakor
Appendix: "Dynamic equilibrium in a competitive credit market: intertemporal contracting as insurance against rationing"
- 282 Arnoud Boot, Anjan V. Thakor and Gregory F. Udell
Credible commitments, contract enforcement problems and banks: intermediation as credibility assurance
- 283 Eduard Ponds
Wage bargaining and business cycles a Goodwin-Nash model
- 284 Prof.Dr. hab. Stefan Mynarski
The mechanism of restoring equilibrium and stability in polish market
- 285 P. Meulendijks
An exercise in welfare economics (II)
- 286 S. Jørgensen, P.M. Kort, G.J.C.Th. van Schijndel
Optimal investment, financing and dividends: a Stackelberg differential game
- 287 E. Nijssen, W. Reijnders
Privatisering en commercialisering; een oriëntatie ten aanzien van verzelfstandiging
- 288 C.B. Mulder
Inefficiency of automatically linking unemployment benefits to private sector wage rates

- 289 M.H.C. Paardekooper
A Quadratically convergent parallel Jacobi process for almost diagonal matrices with distinct eigenvalues
- 290 Pieter H.M. Ruys
Industries with private and public enterprises
- 291 J.J.A. Moors & J.C. van Houwelingen
Estimation of linear models with inequality restrictions
- 292 Arthur van Soest, Peter Kooreman
Vakantiebestemming en -bestedingen
- 293 Rob Alessie, Raymond Gradus, Bertrand Melenberg
The problem of not observing small expenditures in a consumer expenditure survey
- 294 F. Boekema, L. Oerlemans, A.J. Hendriks
Kansrijkheid en economische potentie: Top-down en bottom-up analyses
- 295 Rob Alessie, Bertrand Melenberg, Guglielmo Weber
Consumption, Leisure and Earnings-Related Liquidity Constraints: A Note
- 296 Arthur van Soest, Peter Kooreman
Estimation of the indirect translog demand system with binding non-negativity constraints

IN 1988 REEDS VERSCHENEN

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Factor screening by sequential bifurcation
- 298 Robert P. Gilles
On perfect competition in an economy with a coalitional structure
- 299 Willem Selen, Ruud M. Heuts
Capacitated Lot-Size Production Planning in Process Industry
- 300 J. Kriens, J.Th. van Lieshout
Notes on the Markowitz portfolio selection method
- 301 Bert Bettonvil, Jack P.C. Kleijnen
Measurement scales and resolution IV designs: a note
- 302 Theo Nijman, Marno Verbeek
Estimation of time dependent parameters in linear models
using cross sections, panels or both
- 303 Raymond H.J.M. Gradus
A differential game between government and firms: a non-cooperative
approach
- 304 Leo W.G. Strijbosch, Ronald J.M.M. Does
Comparison of bias-reducing methods for estimating the parameter in
dilution series
- 305 Drs. W.J. Reijnders, Drs. W.F. Verstappen
Strategische bespiegelingen betreffende het Nederlandse kwaliteits-
concept
- 306 J.P.C. Kleijnen, J. Kriens, H. Timmermans and H. Van den Wildenberg
Regression sampling in statistical auditing
- 307 Isolde Woittiez, Arie Kapteyn
A Model of Job Choice, Labour Supply and Wages
- 308 Jack P.C. Kleijnen
Simulation and optimization in production planning: A case study
- 309 Robert P. Gilles and Pieter H.M. Ruys
Relational constraints in coalition formation
- 310 Drs. H. Leo Theuns
Determinanten van de vraag naar vakantiereizen: een verkenning van
materiële en immateriële factoren
- 311 Peter M. Kort
Dynamic Firm Behaviour within an Uncertain Environment
- 312 J.P.C. Blanc
A numerical approach to cyclic-service queueing models

- 313 Drs. N.J. de Beer, Drs. A.M. van Nunen, Drs. M.O. Nijkamp
Does Morkmon Matter?
- 314 Th. van de Klundert
Wage differentials and employment in a two-sector model with a dual labour market
- 315 Aart de Zeeuw, Fons Groot, Cees Withagen
On Credible Optimal Tax Rate Policies
- 316 Christian B. Mulder
Wage moderating effects of corporatism
Decentralized versus centralized wage setting in a union, firm, government context
- 317 Jörg Glombowski, Michael Krüger
A short-period Goodwin growth cycle
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The optimal design of rotating panels in a simple analysis of variance model
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- 320 Th. van de Klundert
Wage Rigidity, Capital Accumulation and Unemployment in a Small Open Economy
- 321 M.H.C. Paardekooper
An upper and a lower bound for the distance of a manifold to a nearby point
- 322 Th. ten Raa, F. van der Ploeg
A statistical approach to the problem of negatives in input-output analysis
- 323 P. Kooreman
Household Labor Force Participation as a Cooperative Game; an Empirical Model
- 324 A.B.T.M. van Schaik
Persistent Unemployment and Long Run Growth
- 325 Dr. F.W.M. Boekema, Drs. L.A.G. Oerlemans
De lokale produktiestructuur doorgelicht.
Bedrijfstakverkenningen ten behoeve van regionaal-economisch onderzoek
- 326 J.P.C. Kleijnen, J. Kriens, M.C.H.M. Lafleur, J.H.F. Pardoel
Sampling for quality inspection and correction: AOQL performance criteria

- 327 Theo E. Nijman, Mark F.J. Steel
Exclusion restrictions in instrumental variables equations
- 328 B.B. van der Genugten
Estimation in linear regression under the presence of heteroskedasticity of a completely unknown form
- 329 Raymond H.J.M. Gradus
The employment policy of government: to create jobs or to let them create?
- 330 Hans Kremers, Dolf Talman
Solving the nonlinear complementarity problem with lower and upper bounds
- 331 Antoon van den Elzen
Interpretation and generalization of the Lemke-Howson algorithm
- 332 Jack P.C. Kleijnen
Analyzing simulation experiments with common random numbers, part II: Rao's approach
- 333 Jacek Osiewalski
Posterior and Predictive Densities for Nonlinear Regression.
A Partly Linear Model Case
- 334 A.H. van den Elzen, A.J.J. Talman
A procedure for finding Nash equilibria in bi-matrix games
- 335 Arthur van Soest
Minimum wage rates and unemployment in The Netherlands
- 336 Arthur van Soest, Peter Kooreman, Arie Kapteyn
Coherent specification of demand systems with corner solutions and endogenous regimes
- 337 Dr. F.W.M. Boekema, Drs. L.A.G. Oerlemans
De lokale produktiestructuur doorgelicht II. Bedrijfstakverkenningen ten behoeve van regionaal-economisch onderzoek. De zeescheepsnieuw-
bouwindustrie
- 338 Gerard J. van den Berg
Search behaviour, transitions to nonparticipation and the duration of unemployment
- 339 W.J.H. Groenendaal and J.W.A. Vingerhoets
The new cocoa-agreement analysed
- 340 Drs. F.G. van den Heuvel, Drs. M.P.H. de Vor
Kwantificering van ombuigen en bezuinigen op collectieve uitgaven
1977-1990
- 341 Pieter J.F.G. Meulendijks
An exercise in welfare economics (III)

- 342 W.J. Selen and R.M. Heuts
A modified priority index for Günther's lot-sizing heuristic under capacitated single stage production
- 343 Linda J. Mittermaier, Willem J. Selen, Jeri B. Waggoner, Wallace R. Wood
Accounting estimates as cost inputs to logistics models
- 344 Remy L. de Jong, Rashid I. Al Layla, Willem J. Selen
Alternative water management scenarios for Saudi Arabia
- 345 W.J. Selen and R.M. Heuts
Capacitated Single Stage Production Planning with Storage Constraints and Sequence-Dependent Setup Times
- 346 Peter Kort
The Flexible Accelerator Mechanism in a Financial Adjustment Cost Model
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De toenemende importantie van het verticale marketing systeem
- 348 P.C. van Batenburg en J. Kriens
E.O.Q.L. - A revised and improved version of A.O.Q.L.
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Multinationalisatie en coördinatie
De internationale strategie van Nederlandse ondernemingen nader beschouwd
- 350 K.A. Bubshait, W.J. Selen
Estimation of the relationship between project attributes and the implementation of engineering management tools
- 351 M.P. Tummers, I. Woittiez
A simultaneous wage and labour supply model with hours restrictions
- 352 Marco Versteijne
Measuring the effectiveness of advertising in a positioning context with multi dimensional scaling techniques
- 353 Dr. F. Boekema, Drs. L. Oerlemans
Innovatie en stedelijke economische ontwikkeling
- 354 J.M. Schumacher
Discrete events: perspectives from system theory
- 355 F.C. Bussemaker, W.H. Haemers, R. Mathon and H.A. Wilbrink
A (49,16,3,6) strongly regular graph does not exist
- 356 Drs. J.C. Caanen
Tien jaar inflatieneutrale belastingheffing door middel van vermogensaftrek en voorraadaftrek: een kwantitatieve benadering

- 357 R.M. Heuts, M. Bronckers
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and service constraints using optimal policy surfaces
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processes

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